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Localization of phonons in two component superlattice with random thicknesses of the layers

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Abstract

Two component superlattice film of $2N$ layers is considered and dimensionally quantised spectrum of phonon-s is found.

The problem of localization of phonon-s in the superlattice with random thicknesses of the layers is investigated. The Landauer resistance of the transport of phonon-s is calculated exactly. For short range disorder the numerical analyses shows, that at frequency ω=0, there is delocalized state and the correlation length index ν is equal to 2.

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1 Introduction

The interest to superlattice structure in condensed matter physics is based on the possibility of manipulation of the physical properties of devices by changing of characteristic lattice parameters.

The growth techniques can be used to prepare specimens consisting of alternating layers of thickness d_1 of constituent A and thickness d_2 of constituent B. Samples can be prepared so that d_1 and d_2 have any value from two or three atomic spacings up to hundred atomic spacings. The entities A and B can be materials with different acoustic, electronic, magnetic properties and one can consider semiconductor, metal, insulator and superconductor constituents, attempting to change values of expectable physical variables in a desirable regime.

The technical advance in fabrication of superlattices motivated an intensive study of various physical properties of these systems, especially electronic and vibration spectra, optical and magnetic properties, e.t.c. The calculations [2, 3] shows that the spectrum of quasi-periodic systems are intermediate between periodic and random ones.

Along with electronic properties study of elastic waves in a bulk superlattices has been a subject of interest in the last decades [18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29]. A one dimensional theory of acoustic vibrations in layered material was given long ago by Rytov [1]. Elastic waves have also been investigated in semi-infinite superlattices [30, 31, 32, 33, 33].

The consideration of the superlattice films instead of the massive ones provides additional opportunities for controlling the elastic and electronic parameters of the superlattices. A sufficiently complete experimental knowledge about the oscillator spectra of binary laminated semiconductor systems is available nowadays [34]. Numerous and generally mutually compatible results about the frequencies of long-wave phonon-s in $InAs - GaSb, Ge - GaAs$ are presented in literature [35].

Together with strongly periodic superlattices the effects of localization and tunnelling of the electrons was studied in short rang disorder superlattices [4, 5, 6, 7]

The problem of localization of electrons in random potential and hopping parameters in low dimensional spaces are in continuous interest of physicists after Andersons remarkable article [8]. Originally Mott and Twose [9] conjectured that all states are localized in $1D$ systems for any degree of disorder. It was argued [10] that in case of full randomness all states are localized in dimensions equal and less of two. However, recent investigations shows [11, 12, 13] that delocalized states can appear in case of correlated disorder.

The aim of present article is twofold. First we found the spectrum of transversal phonon-s in two component superlattice film with boundaries with arbitrary finite number of slices. The results, obtained here, are also applicable for longitudinal waves, which moves in the transversal to the layers direction. Second, we consider random distribution of the thicknesses of the superlattice compounds and calculate the Landauer resistance [14] of the acoustic phonon-s exactly. The continuous model is used, in which the layers are considered as a macroscopic elastic bodies.

2 Notations and equation of motion in the bulk

We follow here the notations and derivations of the book [36].

Let the vector of elastic displacement (deformation) of the material at the space point $\vec{x}$ is $\vec{u}(\vec{x})$. For small variations ($\vec{u} << 1$) we can derive the strain tensor (tensor of deformation) as

$$u_{ik} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x^k} + \frac{\partial u_k}{\partial x^i}\right) \qquad i,k = 1,2,3. \tag{1}$$

The response of the free energy F to variation of deformation tensor defines the stress tensor σ_{ik} as

$$dF = -SdT + \sigma_{ik} du_{ik}, \tag{2}$$

According to Hooke's law, for small deformations u_{ik} around equilibrium, we can restrict ourself to consider only quadratic in u_{ik} terms in series expansion of F. Define

$$dF = \left[K u_{ll}\sigma_{ik} + 2\mu\left(u_{ik} - \frac{1}{3}u_{ll}\delta_{ik}\right)\right] du_{ik}, \tag{3}$$

which defines the following stress tensor

$$\sigma_{ik} = K u_{ll} \sigma_{ik} + 2\mu \left(u_{ik} - \frac{1}{3} \delta_{ik} u_{ll} \right), \tag{4}$$

where K is the modulus of compression and μ is the modulus of rigidity. $K > 0$ and $\mu > 0$. One can express stress σ_{ik} in terms of Young's modulus $E = \frac{3k\mu}{3k+\mu}$ and Poisson's coefficient $\sigma = \frac{1}{2} \frac{3k-3\mu}{3k+\mu}$ as follows

$$\sigma_{ik} = \frac{E}{1+\sigma} \left(u_{ik} + \frac{\sigma}{1-2\sigma} u_{ll} \delta_{ik} \right). \tag{5}$$

Then, the equation of motion for the displacement u_i is simply

$$\rho \ddot{u}_i = \frac{\partial \sigma_{ik}}{\partial x_k} \tag{6}$$

where $F_i = \frac{\partial \sigma_{ik}}{\partial x_k}$ is the force, which acts on unite volume around $\vec{x}$ with density of matter $\rho(\vec{x})$. After substituting (4) into (6) we can define the equation for $u_i(x)$ in a bulk.

$$\rho \ddot{u}_i = \frac{E}{2(1+\sigma)} \Delta u_i + \frac{E}{2(1+\sigma)(1-2\sigma)} \mathrm{grad} \frac{\partial u_i}{\partial x^i}. \tag{7}$$

For the transversal waves

$$\mathrm{div} \vec{u} = 0, \tag{8}$$

and from the (7) we can easily get

$$\frac{\partial^2 u_i}{\partial t^2} - c_t^2 \Delta u_i = 0, \tag{9}$$

with

$$c_t^2 = \frac{E}{2\rho(1+\sigma)}. \tag{10}$$

For the longitudinal waves

$$\Delta \vec{u} = \mathrm{grad}(\mathrm{div} \vec{u}) \tag{11}$$

the equation (7) reduces to the following equation

$$\frac{\partial^2 u_i}{\partial t^2} - c_e^2 \Delta u_i = 0 \tag{12}$$

where c_l is the longitudinal velocity

$$c_l^2 = \frac{E(1-\sigma}{\rho(1+\sigma)(1-2\sigma)}. \tag{13}$$

3 Boundary conditions in the superlattice film, Transfer Matrix and the spectrum

Let's consider superlattice, elementary cells of which consists of two layers of various materials A and B with the thickness d_1, d_2 and modulus of rigidity μ_1, μ_2 (fig.1). The number of pairs in the film is N.

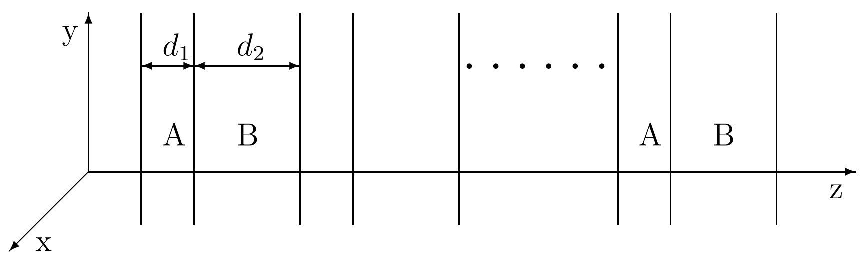

Fig.1.

We consider transversal elastic waves (div$\vec{u} = 0$) propagating inside superlattice in arbitrary direction. It can be shown, that all results are reproducible for longitudinal waves if they propagates in transversal to layers direction.

Lets choose coordinate system such, that waves are propagating in the (x, y) plane (x represents transversal to layers direction), with the wave vectors $(k_1, q_1, 0)$ and $(k_2, q_2, 0)$ in the A and B materials correspondingly. Without loose of generality one can take $u_x = u_y = 0$.

The solutions of the wave equations (9) with frequency ω, which full-fills transversality condition div$\vec{u} = 0$, is the superposition of forward - and a backward-travelling waves

$$u_{2n-1} = \left(c_{2n-1}e^{ik_1x} + \bar{c}_{2n-1}e^{-ik_1x}\right)e^{i(qy-wt)},$$
$$u_{2n} = \left(c_{2n}e^{ik_2x} + \bar{c}_{2n}e^{-ik_2x}\right)e^{i(qy-wt)}, \qquad n = 1\cdots N, \tag{14}$$

where for $k_i, i = 1, 2$ we have

$$k_i^2 + q^2 = \frac{\omega^2}{c_{it}^2} \tag{15}$$

In formulas (14) $2n$(correspondingly$2n-1$) numerates the layers of B(or A) type and $c_{1t}(c_{2t})$ are the velocities of sound in that materials.

We should now impose the boundary conditions on the displacements u_{2n} and u_{2n-1}.

Let's consider now free boundaries of the film, which means the use of Neumann boundary conditions

$$\partial_x u_1 = \partial_x u_{2N} = 0. \tag{16}$$

On the boundary of the A and B layers one should use the continuity condition for the displacements

$$u_{2n} = u_{2n-1}, \tag{17}$$

as well for the forces

$$F_i = \sigma_{ik} ds_k. \tag{18}$$

In the equation (18) $ds_k = n_k ds$-is the normal vector to the boundary and equal to small area in modulo. Hence we have

$$\sigma^1_{ik} n_k = \sigma^2_{ik} n_k. \tag{19}$$

On the boundaries of the film the forces are equal to zero

$$\sigma_{ik} n_k = 0 \tag{20}$$

By use of expression (5) for σ_{ik} from the boundary conditions (16)-17) and (19-20) one can easily obtain following set of equations for the displacements u

$$\begin{aligned} u^{2N}_x &= u^1_x = 0, \\ \mu_2 u^{2n}_x &= \mu_1 u^{2n-1}_x, \\ u^{2n} &= u^{2n-1} \qquad n = 1, 2, \cdots N. \end{aligned} \tag{21}$$

This equations transforms into the following equations for the coefficients of the forward and backward travelling waves

$$c_{2N} e^{ik_2(d_1+d_2)n} - \bar{c}_{2N} e^{-ik_2(d_1+d_2)n} = 0$$

$$\mu_2 k_2 c_{2n} e^{ik_2[(d_1+d_2)(n-1)+d_1]} - \mu_2 k_2 \bar{c}_{2n} e^{-ik_2[(d_1 n+d_2(n-1)]} =$$
$$= \mu_1 k_1 c_{2n-1} e^{ik_1[d_1 n+d_2(n-1)]} - \mu_1 k_1 \bar{c}_{2n-1} e^{-ik_1[d_1 n+d_2(n-1)]}$$
$$c_{2n} e^{ik_2[d_1 n+d_2(n-1)]} + \bar{c}_{2n} e^{-ik_2[d_1 n+d_2(n-1)]} =$$
$$= c_{2n-1} e^{ik_1[d_1 n+d_2(n-1)]} + \bar{c}_{2n-1} e^{-ik_1[d_1 n+d_2(n-1)]} \tag{22}$$
$$\vdots$$
$$c_1 - \bar{c}_1 = 0$$

We will solve this set of linear equations by use of transfer matrix method [18].

Let's define now

$$\psi_{2n} = \begin{pmatrix} c_{2n} \\ \bar{c_{2n}} \end{pmatrix}. \tag{23}$$

Then the half of the set of equations (23)can be reformulated as follows

$$A_{2n}\psi_{2n} = B_{2n-1}\psi_{2n-1}, \tag{24}$$

with

$$A_{2n} = \begin{pmatrix} e^{ik_2(nd_1+(n-1)d_2)}, & -e^{-ik_2(nd_1+(n-1)d_2)} \\ e^{ik_2(nd_1+(n-1)d_2)}, & e^{-ik_2(nd_1+(n-1)d_2)} \end{pmatrix}, \tag{25}$$

and

$$B_{2n-1} = \begin{pmatrix} \frac{\mu_1 k_1}{\mu_2 k_2} e^{ik_1(nd_1+(n-1)d_2)}, & -\frac{\mu_1 k_1}{\mu_2 k_2} e^{-ik_1(nd_1+(n-1)d_2)} \\ e^{ik_1(nd_1+(n-1)d_2)}, & e^{-ik_1(d_1 n+(n-1)d_2)} \end{pmatrix}. \tag{26}$$

Equation (24) can be rewritten as

$$\psi_{2n} = A_{2n}^{-1}\, B_{2n-1}\, \psi_{2n-1}. \tag{27}$$

Similarly, the other half of the equations (22) looks as

$$\psi_{2n-1} = A_{2n-1}^{-1}\, B_{2n-2}\, \psi_{2n-2}, \tag{28}$$

with

$$A_{2n-1} = \begin{pmatrix} e^{ik_1((n-1)d_1+(n-1)d_2)}, & -e^{-ik_1((n-1)d_1+(n-1)d_2)} \\ e^{ik_1((n-1)d_1+(n-1)d_2)}, & e^{-ik_1((n-1)d_1+(n-1)d_2)} \end{pmatrix}, \tag{29}$$

and

$$B_{2n-2} = \begin{pmatrix} \frac{\mu_2 k_2}{\mu_1 k_1} e^{ik_2[(n-1)d_1+(n-1)d_2]}, & -\frac{\mu_2 k_2}{\mu_1 k_1} e^{-ik_2[(n-1)d_1+(n-1)d_2]} \\ e^{ik_2[(n-1)d_1+(n-1)d_2]}, & e^{-ik_2[(n-1)d_1+(n-1)d_2]} \end{pmatrix}. \tag{30}$$

Recursion equations (27) and (28) allows us to connect ψ_{2n} with ψ_1 in a following way

$$\psi_{2n} = A_{2n}^{-1}\, B_{2n-1}\, A_{2n-1}^{-1}\, B_{2n-2} \cdots A_2^{-1}\, B_1\, \psi_1. \tag{31}$$

Let's now define the Transfer Matrices as

$$T_1 = B_{2n-1}\, A_{2n-1}^{-1} = \begin{pmatrix} \frac{\mu_1 k_1}{\mu_2 k_2} \cos k_1 d_1 & i \frac{\mu_1 k_1}{\mu_2 k_2} \sin k_1 d_1 \\ i \sin k_1 d_1 & \cos k_1 d_1 \end{pmatrix} \tag{32}$$

and

$$T_2 = B_{2n}\, A_{2n}^{-1} = \begin{pmatrix} \frac{\mu_2 k_2}{\mu_1 k_1} \cos k_2 d_2 & i \frac{\mu_2 k_2}{\mu_1 k_1} \sin k_2 d_2 \\ i \sin k_2 d_2 & \cos k_2 d_2 \end{pmatrix}. \tag{33}$$

Then the equation (31) becomes

$$\psi_{2n} = B_{2n}^{-1}\, T^n\, A_1\, \psi_1 = U \psi_1 \tag{34}$$

where

$$T = T_1 T_2 = \begin{pmatrix} \cos k_1 d_1 \cos k_2 d_2 - & i \cos k_1 d_1 \sin k_2 d_2 + \\ -\frac{\mu_1 k_1}{\mu_2 k_2} \sin k_1 d_1 \sin k_2 d_2, & +\frac{i\mu_1 k_1}{\mu_2 k_2} \sin k_1 d_1 \cos k_2 d_2 \\ & \\ i \sin k_1 d_1 \cos k_2 d_2 + & -\frac{\mu_2 k_2}{\mu_1 k_1} \sin k_1 d_1 \sin k_2 d_2 + \\ +\frac{i\mu_1 k_1}{\mu_2 k_2} \cos k_1 d_1 \sin k_2 d_2 & + \cos k_1 d_1 \cos k_2 d_2 \end{pmatrix} \tag{35}$$

The first equation of (22) for $n = N$ can be written as

$$\bar{\psi}_{2N} \psi_{2N} = 0, \tag{36}$$

where

$$\bar{\psi}_{2N} = \left(e^{ik_2(Nd_1+Nd_2)}, \quad -e^{-ik_2(Nd_1+Nd_2)} \right). \tag{37}$$

For the another boundary of the superlattice film, where $n = 1$ we have

$$c_1 = \bar{c}_1 = c, \tag{38}$$

which means that

$$\psi_1 = c \begin{pmatrix} 1 \\ 1 \end{pmatrix}. \tag{39}$$

From the equations (34) and (36) we can obtain following equation for the spectrum of transversal phonon-s in the superlattice film of $2N$ layers

$$Tr\left[CT^N\right] = 0, \tag{40}$$

where

$$C^\alpha_\beta = (A_1\psi_1)_\beta(\bar{\psi}_{2N}B^{-1}_{2N})^\alpha = \begin{pmatrix} 0 & 0 \\ 2\frac{\mu_1 k_1}{\mu_2 k_2} & 0 \end{pmatrix}. \tag{41}$$

To proceed further we need to calculate the $N-1$ degree of the Transfer Matrix, which can be achieved simply by diagonalizing T. Obviously

$$T^N = W^{-1} \begin{pmatrix} \lambda^N, & 0 \\ 0, & \bar{\lambda}^N \end{pmatrix} W, \tag{42}$$

where λ and $\bar{\lambda}$ are eigenvalues of T, and W is the diagonalizing matrix. One easily can find the eigenvalues or the Transfer Matrix T as

$$\begin{aligned} \lambda = e^{\pm i\Theta} \quad &= \quad \left(\cos k_1 d_1 \cos k_2 d_2 - \frac{1}{2}\left(\frac{\mu_1 k_1}{\mu_2 k_2} + \frac{\mu_2 k_2}{\mu_1 k_1}\right)\sin k_1 d_1 \sin k_2 d_2\right) \pm \\ &\pm \quad i\sqrt{1 - \left(\cos k_1 d_1 \cos k_2 d_2 - \frac{1}{2}\left(\frac{\mu_1 k_1}{\mu_2 k_2} + \frac{\mu_2 k_2}{\mu_1 k_1}\right)\sin k_1 d_1 \sin k_2 d_2\right)^2}, \end{aligned} \tag{43}$$

where

$$\cos\Theta = \cos k_1 d_1 \cos k_2 d_2 - \left(\frac{\mu_1 k_1}{\mu_2 k_2} + \frac{\mu_2 k_2}{\mu_1 k_1}\right)\sin k_1 d_1 \sin k_2 d_2. \tag{44}$$

Further, a simple calculations shows, that equation (40) reduces to

$$Im\lambda^N = 0, \tag{45}$$

which means that

$$\Theta = \pi\frac{Q}{N}, \quad Q = 1 \cdots N. \tag{46}$$

Finally we obtain following equation for the spectrum of transversal phonon-s

$$\cos k_1 d_1 \cos k_2 d_2 - \left(\frac{\mu_1 k_1}{\mu_2 k_2} + \frac{\mu_2 k_2}{\mu_1 k_1}\right) \sin k_1 d_1 \sin k_2 d_2 = \pm \cos \pi \frac{Q}{N} \tag{47}$$

where

$$k_i^2 = \frac{\omega^2}{c_i^2} - q^2 \tag{48}$$

We see that this equation is coinciding with the equation for the spectrum of phonon-s in the bulk [1, 18], but the momentums in perpendicular to the layers direction are quantised due to dimensional restriction of the film.

4 The Landauer resistance of phonon-s in the superlattice with random distribution of thicknesses of the layers

The problem of elastic waves in superlattice is essentially one dimensional. One dimensional problems are especially attractive because of their possible exact integrability. In the article by Erdos and Herndon [15] the problem of the transport of particles in the one dimensional space for a wide class of disorders was considered in the Transfer Matrix approach and general results were obtained. It was proved that that Transfer Matrix of the one dimensional problem belongs to $SL(2, R)$ group and randomness can be exactly taken into account for such quantities as Landauer resistance [14].

Some exact results for Kronig-Penney model in case of non-diagonal disorder by other methods was obtained in [17].

It is easy to see from the formulas (35) for the Transfer Matrix T, here we also have a representative of the $SL(2, R)$ group. One can make a link between transfer Matrices of the Kronig-Penney model and phonon-s in the superlattice.

Following [14] and [15] let's define resistance as a ratio of reflection to transmission coefficients, which, by use of formula (34), is equal to

$$\rho = \frac{1 - |\tau|^2}{|\tau|^2} = U_{12} U_{12}^* = U_2^1 (U^+)_1^2, \tag{49}$$

where U_2^1 is the 1,2 matrix element of the evolution matrix U

$$U = B_{2N}^{-1}(I_1 T_2)^N A_1. \tag{50}$$

We are going to consider random distribution of thicknesses of the layers and take the average of Landauer resistance. For further convenience we will normalise $T_1(T_2)$ Transfer Matrices on order to have a unit determinant. It will not change the equation (50) because the normalisation factors for T_1 and T_2 cancels each other. Hence we will consider

$$T_{2i} = \begin{pmatrix} (\frac{\mu_2 k_2}{\mu_1 k_1})^{1/2} \cos k_2(x_{2i} - x_{2i-1}) & i(\frac{\mu_2 k_2}{\mu_1 k_1})^(1/2) \sin k_2(x_{2i} - x_{2i-1}) \\ i(\frac{\mu_1 k_1}{\mu_2 k_2})^{1/2} \sin k_2(x_{2i} - x_{2i-1}) & (\frac{\mu_1 k_1}{\mu_2 k_2})^{1/2} \cos k_2(x_{2i} - x_{2i-1}) \end{pmatrix} \tag{51}$$

for the even slices. The similar expression for odd slices T_{2i-1} can be found simply by permuting variables k and μ for 1 and 2.

Now let us analyse the direct product of the evolution matrices, the $(U \otimes U^+)_{2,1}^{1,2}$ matrix element of which defines Landauer resistance. For this purpose we should calculate first the simplest constituent block of that expression, namely direct product $T_i \otimes T_i^+$ of U_i-s. In the article [15] it was demonstrated, that this direct product can be represented as $1 \oplus$ (3x3) =4 x 4 matrix. It happened because of the fact, that T_i matrices are a spinor representations of the $SL(2, R)$, hence, the direct product of two 1/2-representations can be expanded as a sum of scalar and vector representations. In the language of the group elements $T \in SL(2, R)$ this expansion looks like

$$(T_i)^{\alpha}_{\alpha'} (T_i^{-1})^{\beta'}_{\alpha} = \frac{1}{2} \delta^{\alpha}_{\beta} \delta^{\beta'}_{\alpha'} - \frac{1}{2} (\sigma^{\mu})^{\beta'}_{\alpha'} \Lambda_i^{\mu\nu} (\sigma^{\nu})^{\alpha}_{\beta}, \tag{52}$$

where

$$\Lambda_i^{\mu\nu} = \frac{1}{2} Tr(T_i \sigma^{\mu} T_i^{-1} \sigma^{\nu}) \tag{53}$$

is the spin-one part of the direct product. But for Landauer resistance we need to calculate $T \otimes T^+$. It is easy to see from the formula (51) that

$$\sigma_1 T^{-1} \sigma_1 = T^+, \tag{54}$$

therefore, by multiplying the the expression (52) in the left and right by σ_1 we will have

$$(T_i)^{\alpha}_{\alpha'}(T_i^+)^{\beta'}_{\beta} = \frac{1}{2}(\sigma_1)^{\alpha}_{\beta}(\sigma_1)^{\beta'}_{\alpha'} - \frac{1}{2}(\sigma^\mu\sigma_1)^{\beta'}_{\alpha'}\Lambda_i^{\mu\nu}(\sigma^\nu\sigma_1)^{\alpha}_{\beta}. \tag{55}$$

Now the calculation of the direct product $U \otimes U^+$ is straightforward. The product $\prod_{i=1}^{2N} T_i$ of T_i-s transforms into product of $\Lambda_i^{\mu\nu}$-s. Finally we will obtain

$$\begin{aligned}(U)^{\alpha}_{\alpha'}(U^+)^{\beta'}_{\beta} &= (B_{2N}^{-1})^{\alpha}_{\gamma}(A_1^+)^{\beta'}_{\delta'}\left[\frac{1}{2}(\sigma_1)^{\gamma}_{\delta}(\sigma_1)^{\delta'}_{\gamma'} - \right.\\ &- \left.\frac{1}{2}(\sigma^\mu\sigma_1)^{\delta'}_{\gamma'}(\prod_{i=1}^{N}\Lambda_{2i-1}\Lambda_{2i})^{\mu\nu}(\sigma^\nu\sigma_1)^{\gamma}_{\delta}\right](A_1)^{\gamma'}_{\alpha'}(B_{2N}^{-1})^{+\delta}_{\beta}.\end{aligned} \tag{56}$$

Substituting this expression, together with the expressions for B_{2N}^{-1} and A_1 (from (30) and (29) correspondingly) into the (49), after some simple algebra for Landauer resistance ρ we will have

$$\begin{aligned}\rho &= \frac{1}{2}(\frac{\mu_1 k_1}{\mu_2 k_2})\left[-1 + (\Lambda^N)^{11}\frac{1}{2}(\frac{\mu_1 k_1}{\mu_2 k_2} + \frac{\mu_2 k_2}{\mu_1 k_1}) + \right.\\ &+ \left. i(\Lambda^N)^{12}\frac{1}{2}(\frac{\mu_1 k_1}{\mu_2 k_2} - \frac{\mu_2 k_2}{\mu_1 k_1})\right],\end{aligned} \tag{57}$$

where $(\Lambda^N)^{11}$ (correspondingly $(\Lambda^N)^{12}$) is the 11(12) matrix element of the matrix $\Lambda = \Lambda_1\Lambda_2$, which is a product of Λ-s of the I and II slices.

The average over any type of random distributions of the parameters of the model can be calculated now exactly. We consider random distribution of thicknesses of the slices, keeping boundaries fixed $x_0 = 0, x_{2N} = L$. We see from the formula (54) that T_i depends only on the thickness of the slice $x_i - x_{i-1}$. The only restriction we have is the condition, that

$$\sum_{i=1}^{2N}\Delta x_i = L. \tag{58}$$

Therefore, the average of the Λ^N, with the probability distribution $g(y)$; $(\int_0^\infty g(y)dy = 1)$, defined in the following way

$$\begin{aligned}\langle\prod_{i=1}^{N}\Lambda_{2i-1}\Lambda_{2i}\rangle &= \int_0^\infty dy_1\dots dy_{2N}g(y_1)\dots g(y_{2N})\delta(\sum_{j=1}^{2N}y_j - L)\prod_{i=1}^{N}\Lambda_{2i-1}(y_{2i-1})\Lambda_{2i}(y_{2i}) = \\ &= \int_{-\infty}^{\infty}dpe^{-ipL}\left(\langle\Lambda_1(p)\rangle\langle\Lambda_2(p)\rangle\right)^N,\end{aligned} \tag{59}$$

where

$$\langle\Lambda_{1,2}(p)\rangle = \int_0^\infty dy e^{ipy} g(y)\Lambda_{1,2}(y). \tag{60}$$

The average Landauer resistance is equal now

$$\begin{aligned}\langle\rho\rangle &= \frac{1}{2}(\frac{\mu_1 k_1}{\mu_2 k_2})\left\{-1+\frac{1}{2}(\frac{\mu_1 k_1}{\mu_2 k_2}+\frac{\mu_2 k_2}{\mu_1 k_1})\int_{-\infty}^{\infty} dp e^{ipL}\left[(\langle\Lambda_1(p)\rangle\langle\Lambda_2(p)\rangle)^N\right]^{11}+\right. \\ &+ \left.\frac{1}{2}(\frac{\mu_1 k_1}{\mu_2 k_2}-\frac{\mu_2 k_2}{\mu_1 k_1})\int_{-\infty}^{\infty} dp e^{ipL}\left[(\langle\Lambda_1(p)\rangle\langle\Lambda_2(p)\rangle)^N\right]^{12}\right\}. \end{aligned} \tag{61}$$

It is obvious, that in a case of homogeneous media (two components of the superlattice are coinciding) we restore the expression for the Landauer resistance of electrons, obtained in [15].

For large sample size ($N >> 1$), as it was argued in [14, 16], the resistance should behave as $e^{N/\xi}$, where ξ is the correlation length. Excitations are localized or not depends on the behaviour of ξ. If at some frequencies correlation length becomes infinite, we have delocalized state and the expression (61) shows, that the answer depends on average value of $\Lambda^{\mu\nu}$. For further analyse let's consider simplest case of the distribution, namely when there is equal probability for slices to have a thickness up to d_i (i=1,2).

$$g(y) = \begin{cases} \frac{1}{d_i}, & 0 < y < d_i \\ 0, & otherwise \end{cases}. \tag{62}$$

We have taken $Al_xGa_{1-x}As$ and $GaAs$ as a components of the superlattice with the parameters [34]

$$\begin{aligned} \mu_1 &= (3.25-0.09x)10^{11} dyn/cm^2, \quad \mu_2 = 3.2510^{11} dyn/cm^2, \\ \rho_1 &= (5.3176-1.6x)g/cm^3, \qquad \rho_2 = 5.3176 g/cm^3, \\ d_1 &= 30\cdot(5.6532+0.0078x)A^o, \qquad d_2 = 10\cdot 5.6532 A^o \end{aligned} \tag{63}$$

and consider waves, propagating in the perpendicular to the layers direction ($\vec{q}=0$).

For large enough N the asymptotics of ρ, and therefore the correlation length ξ, defined by the closest to unity eigenvalues of $\langle\Lambda_1\rangle\langle\Lambda_2\rangle$. If it is λ, then

$$\xi(\omega) \sim 1/\ln\lambda(\omega). \tag{64}$$

Numerical calculations by use of Mathematica shows, that $\lambda(\omega = 0) = 1$, hence $\xi \to \infty$.

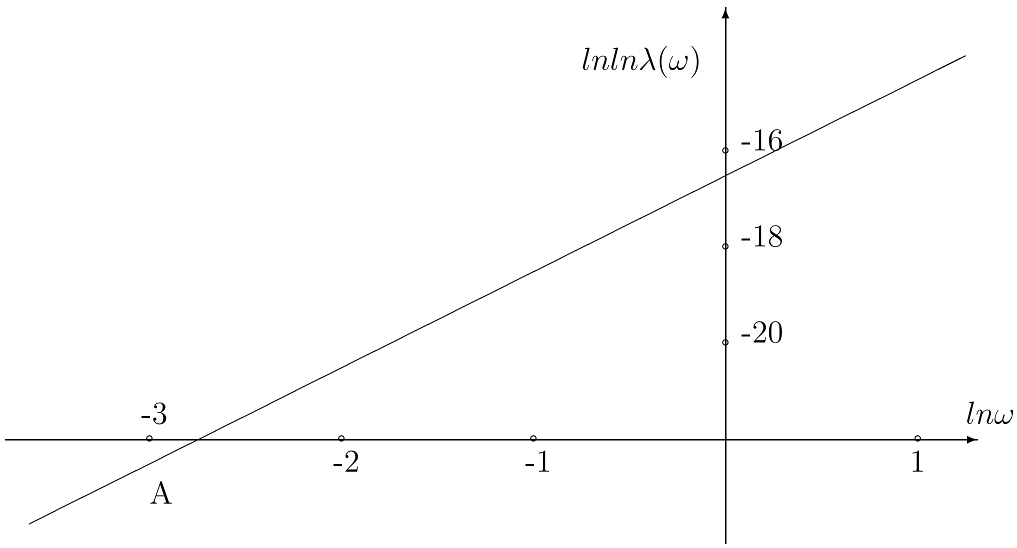

Fig.2

This result is easy to understand, $\omega = 0$ means that we have constant displacement $\vec{u}$, which simply is the sift of the all sample. The correlation index ν from $\xi \sim \omega^{-\nu}$, defined as a slop of the plot of $lnln\lambda(\omega)$ versus $ln\omega$ is presented in Fig.2 and it appeared that ν=2. All other states are localized.

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